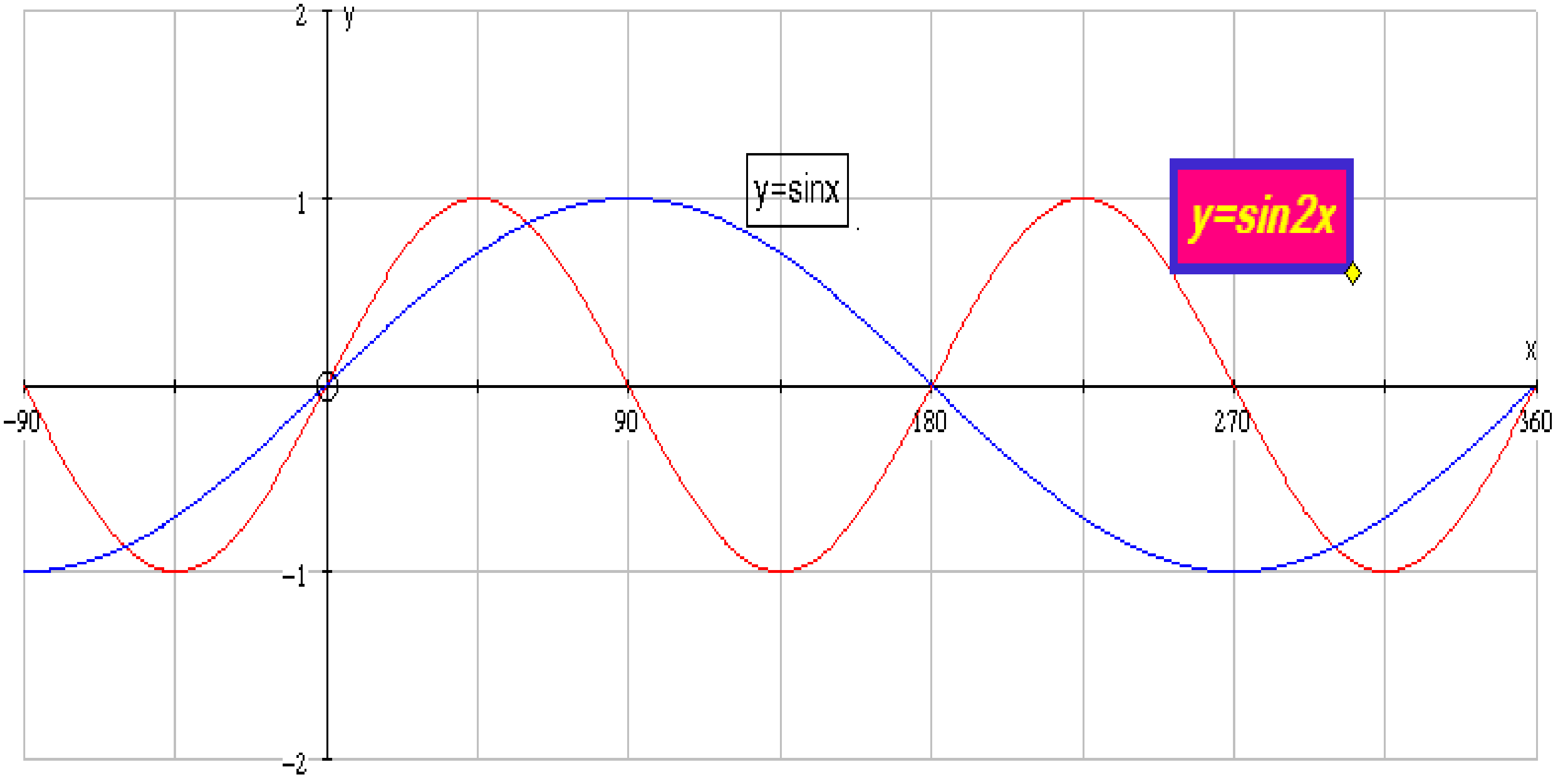
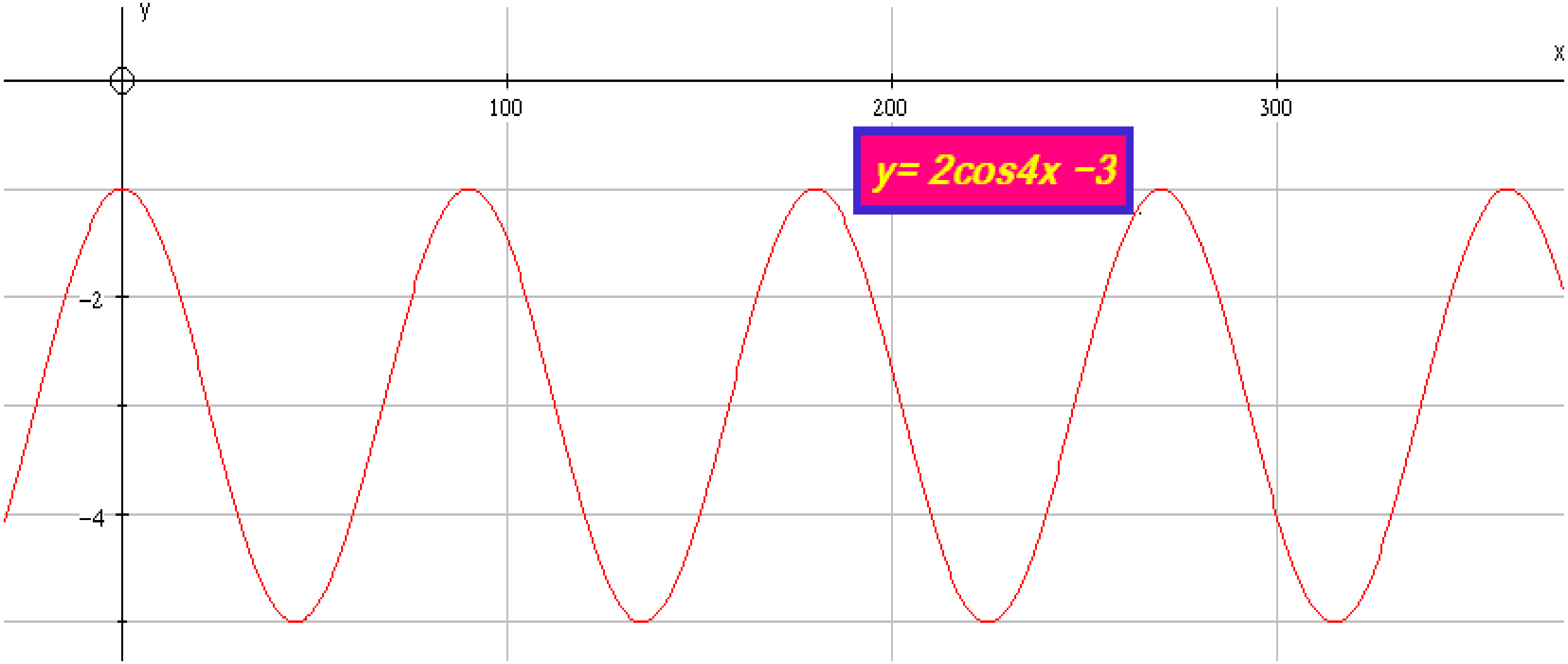
Drawing Trigonometric Graphs.



Combining The Effects.



We are now going to draw more complex trigonometric graphs like the one shown above, by considering what each part of the equation does to the graph of the equation.

Example 1.

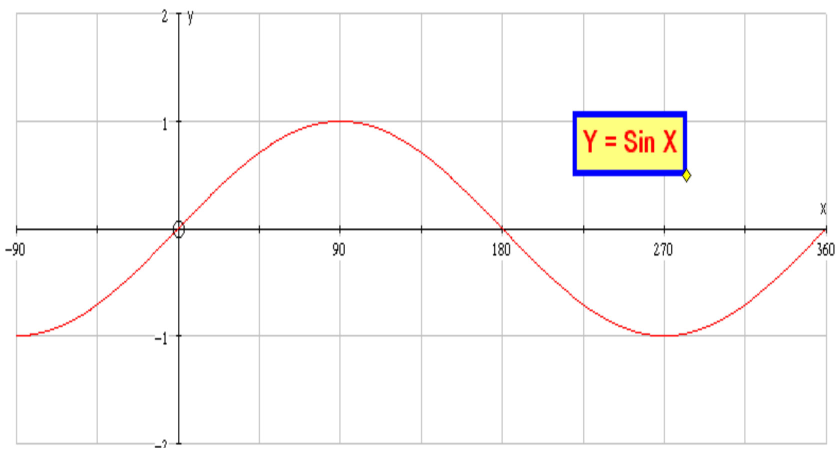
Draw the graph of :

$$y = 4\sin 2x + 3$$

<u>Solution.</u>

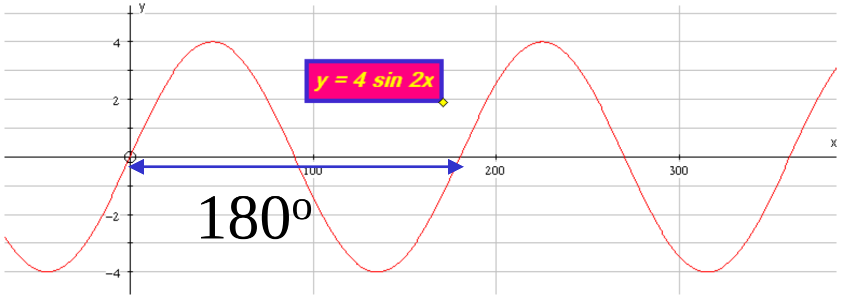
Draw the graph of :

$y = \sin x$

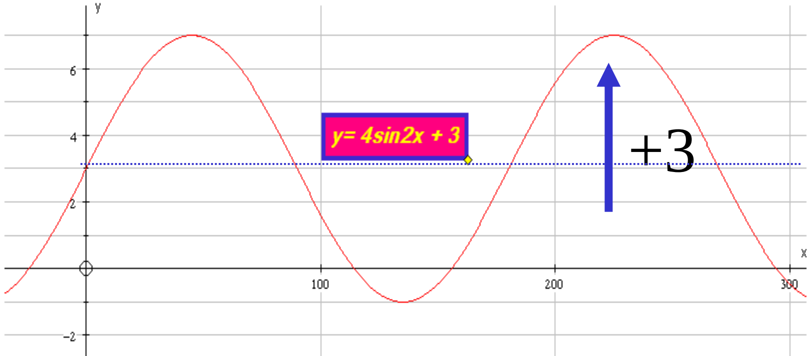


$y = 4 \sin x$

$y = 4 \sin 2x$



$y = 4\sin 2x + 3$



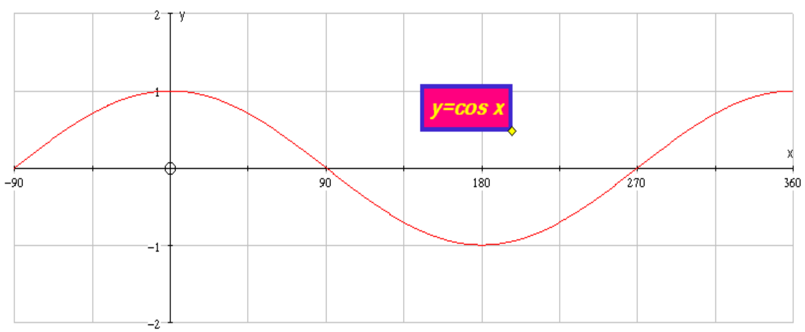
Example 2

Draw the graph y = 2 – 6 cos 5x

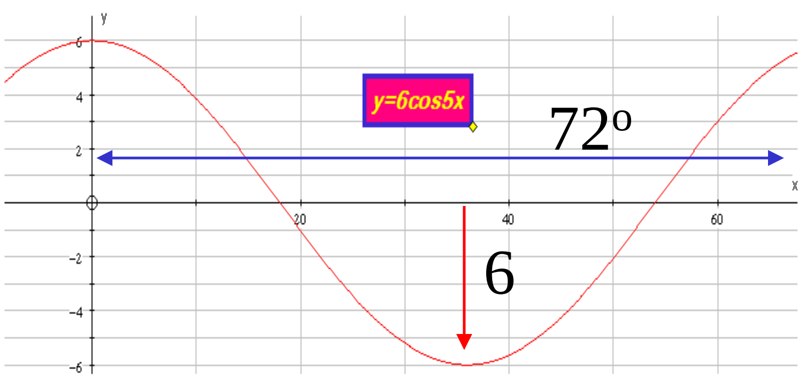
Solution.

Draw the graph of :

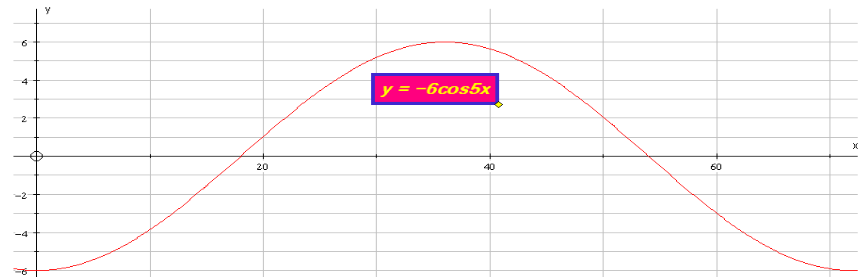
y = cos x



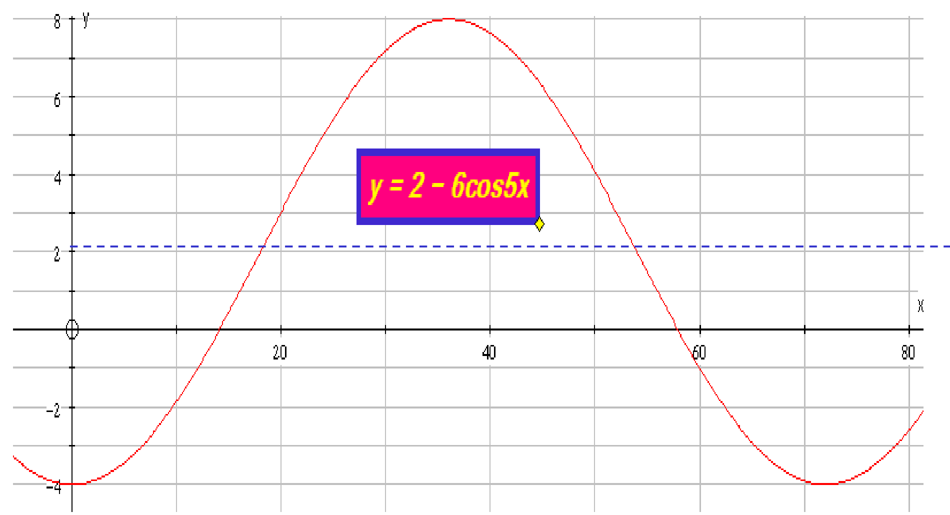
y= 6cos5x



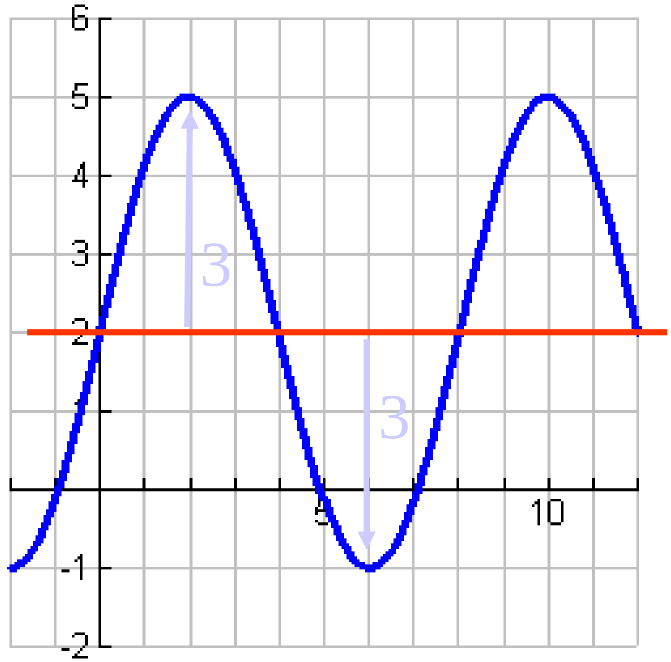
y = - 6cos5x



y = 2 – 6 cos 5x



Example # 1



- To find the amplitude

$$a = \frac{5 - -1}{2} = 3$$

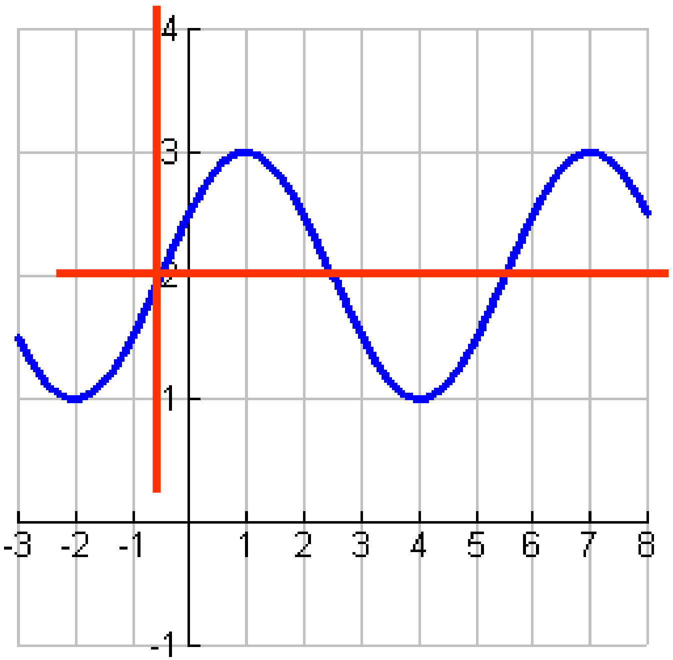
- To find the axis of the wave

$$x = \frac{5 + -1}{2} = 2$$

- To verify the amplitude, what is the vertical distance from the axis of the wave to the peak or valley?

y = 3 sin (x) + 2

Example # 2

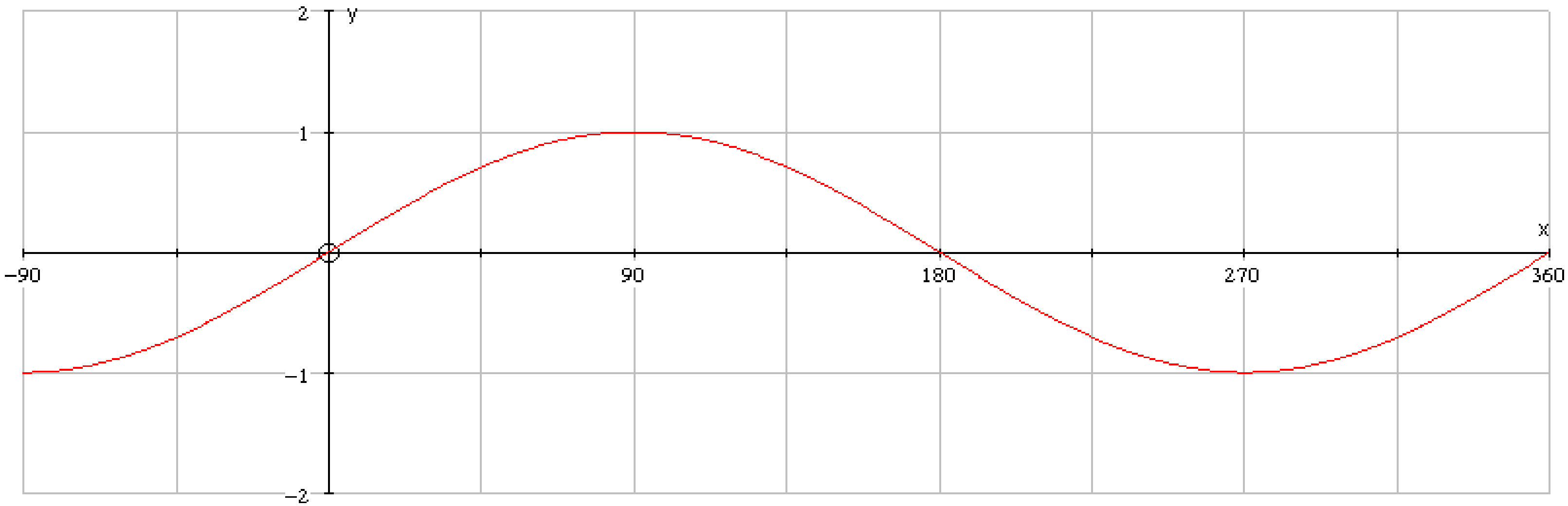


- Axis of wave = 2
- Move the y-axis to the left ½ unit.
- Now it's a sine wave!
- Equation?

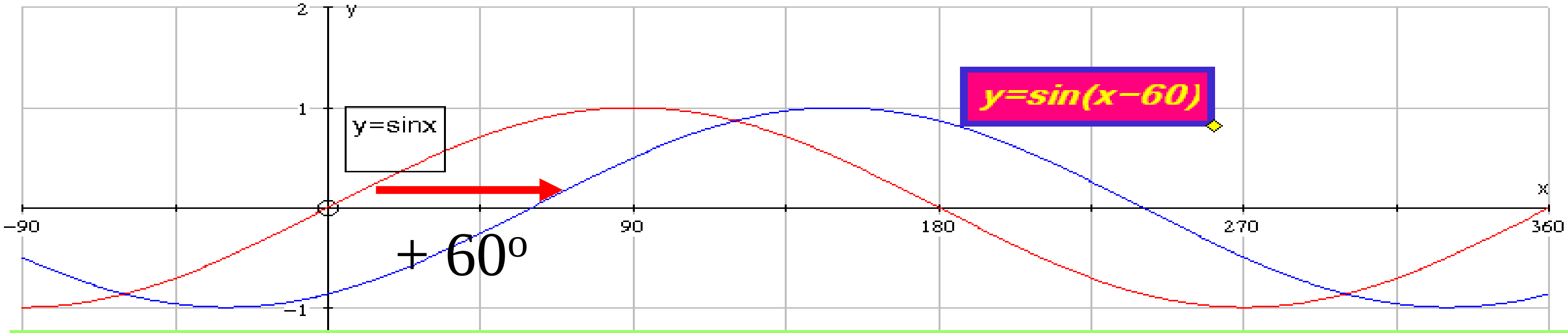
$y = 1 \sin (x - \frac{1}{2}) + 2$

Creating A Phase Shift.

Shown below is the graph of $y = \sin x^{o}$

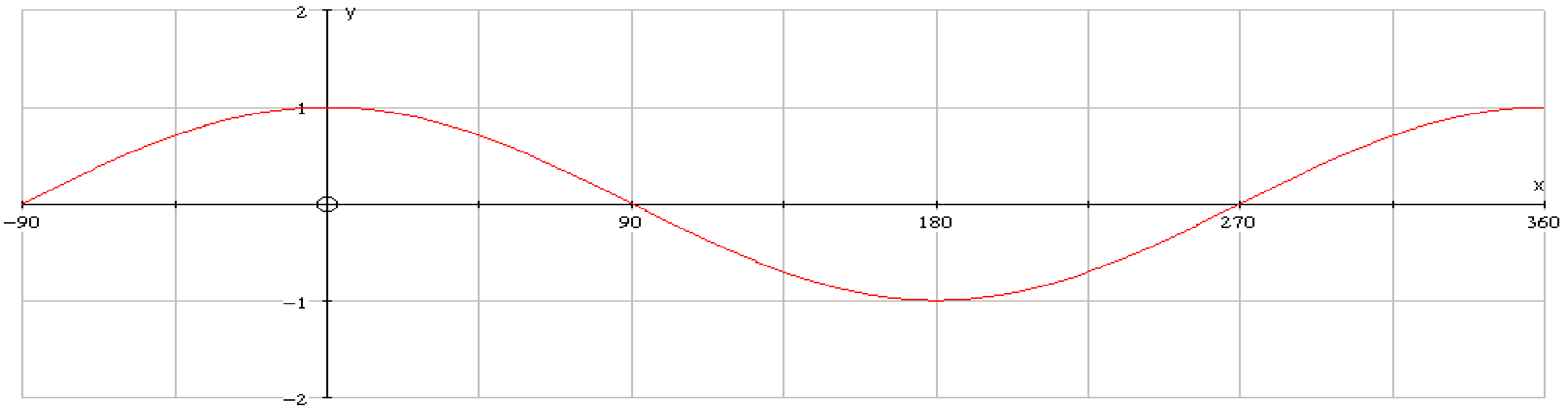


Now compare it with the graph of $y = \sin(x - 60^{o})$

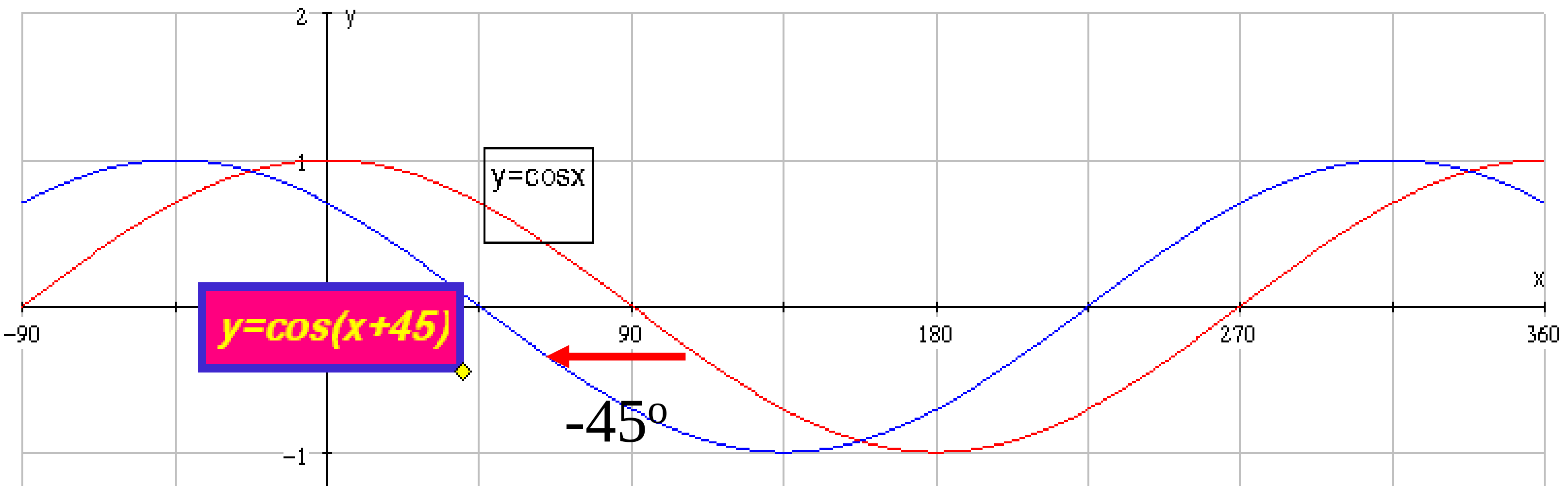


The graph is translated 60^{o} to the right parallel to the x axis.

Shown below is the graph of y = cos x.



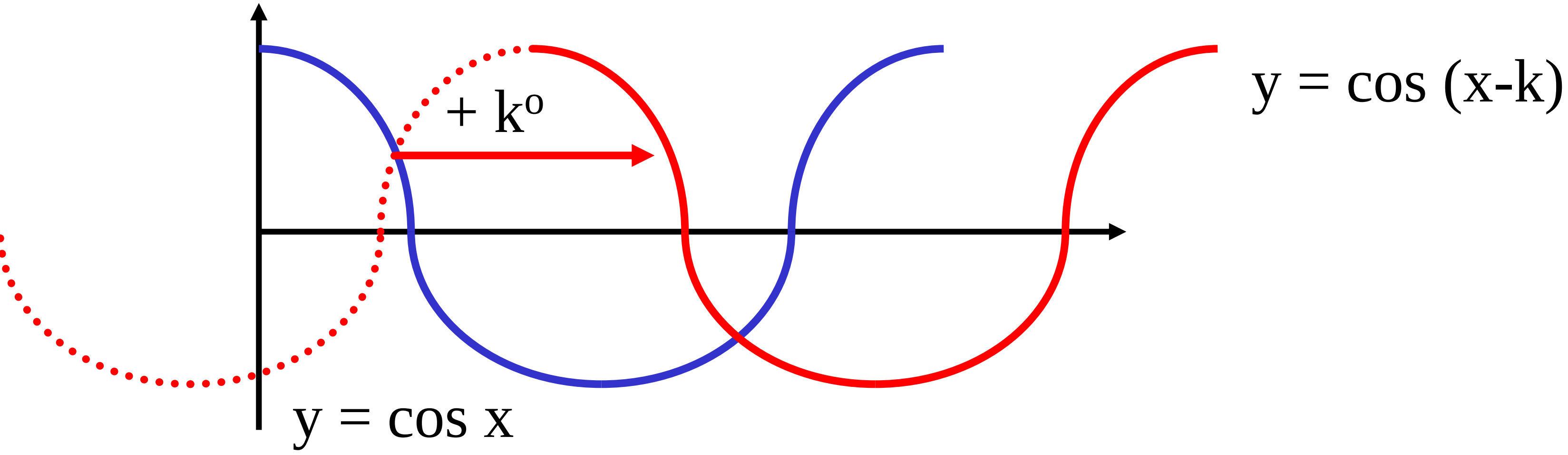
Now compare it to the graph of y = cos (x + 45°)



The graph is translated 45° to the left parallel to the x axis.

From the previous examples we can now see that the equation:

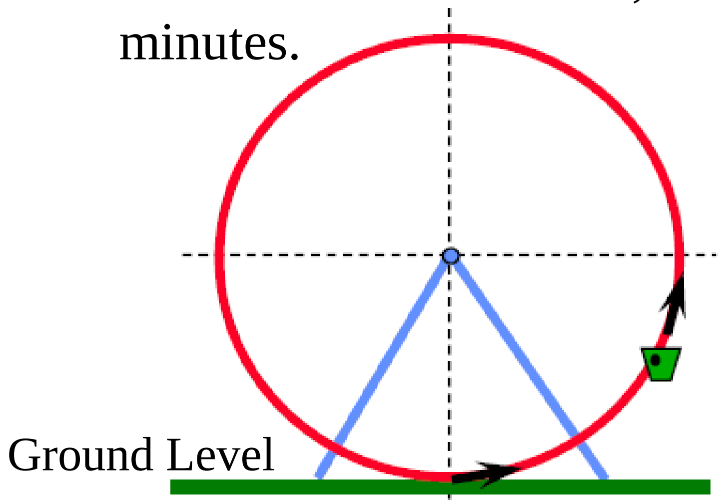
$y = \cos(x - k)$ & $y = \sin(x - k)$ translates the graph k^o to the right parallel to the x axis.



From the previous examples we can now see that the equation:

$y = \cos(x + k)$ & $y = \sin(x + k)$ translates the graph k^o to the left parallel to the x axis.

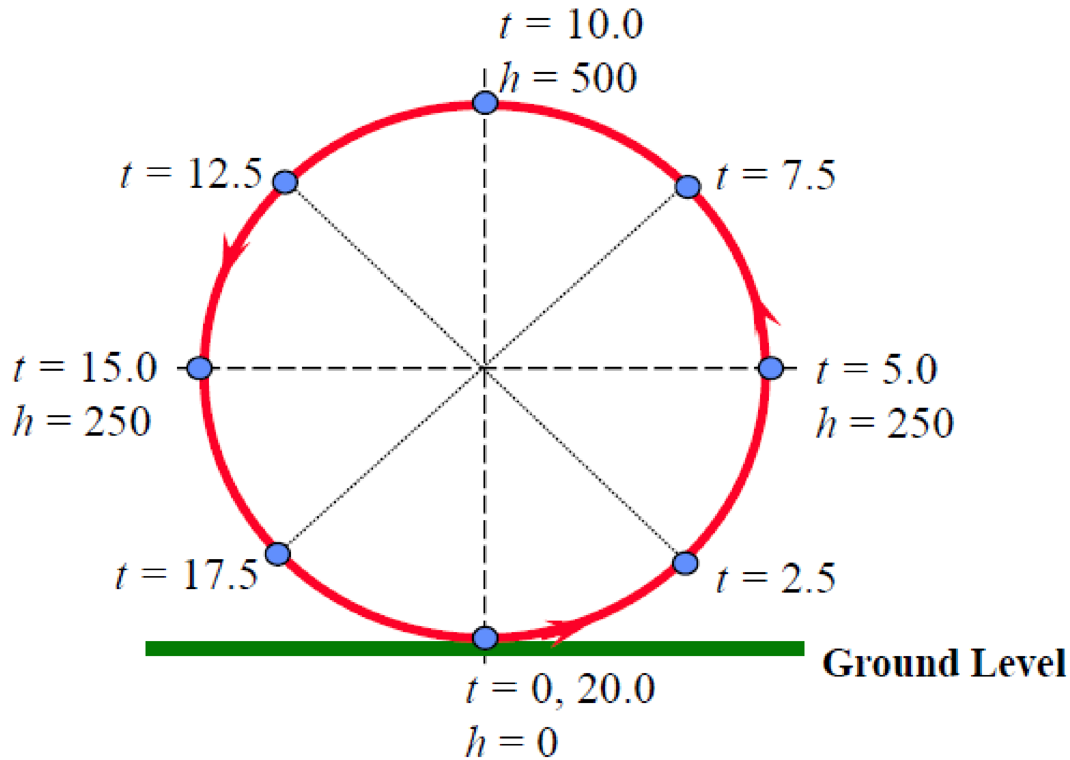
To celebrate the new millennium, British Airways announced in 1996 its plans to fund construction of the world's largest Ferris wheel. The wheel was designed to measure approximately 500 feet in diameter and to carry up to 800 passengers in 32 capsules. The wheel was designed to turn continuously and to be slow enough for people to hop on and off while it turns, completing a single rotation once every 20 minutes.



Suppose you board this Ferris wheel at ground level. Let *t* represent the time since you boarded (in minutes) and let *h* represent your height above ground level (in feet).

Complete the table:

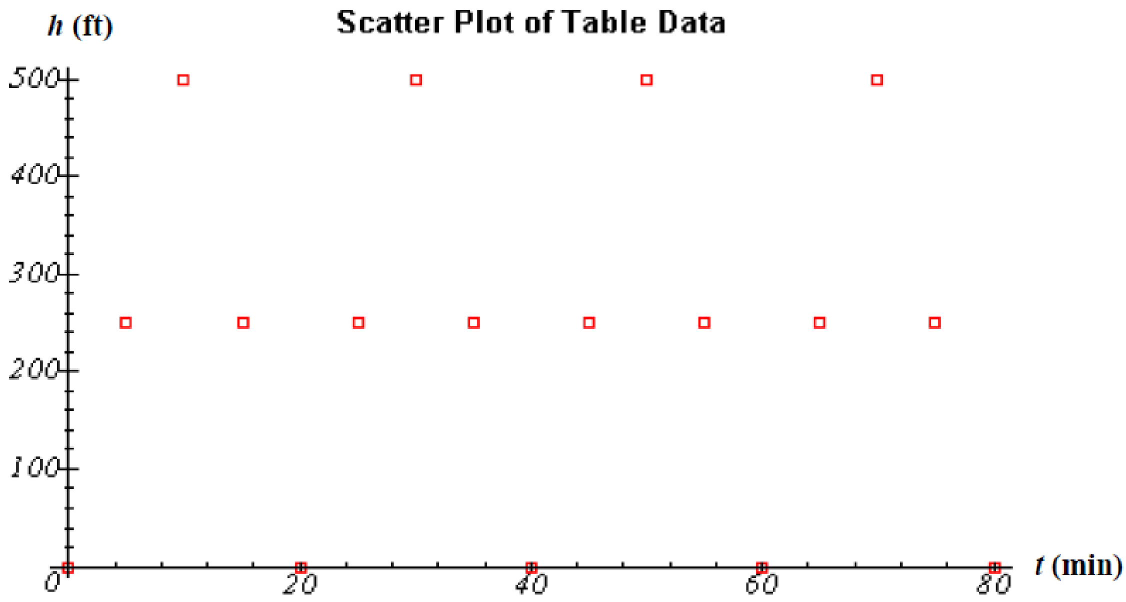
t (min)	0	5	10	15	20	25	30	35	40
h (feet)	0								0



t	0	5	10	15	20	25	30	35	40
h	0	250	500	250	0	250	500	250	0

- Generalize the behavior of the values of *h*. Is there a pattern? Extend your table to include four of these repeated cycles.
- What would this look like if graphed?

Draw and label a scatter plot of *h* versus *t*.



Observations??

•What would be a good description of the <u>shape</u> of the graph if the data points were connected with a smooth curve?

•What natural phenomena have this shape?

•What characteristics will we be interested in?

Since the Ferris wheel data is periodic, we can use a periodic function to model the relationship between *h* and *t*.

The sine function is typically used to find the equation of periodic functions; however, the cosine graph is also used.

We need some more terminology…

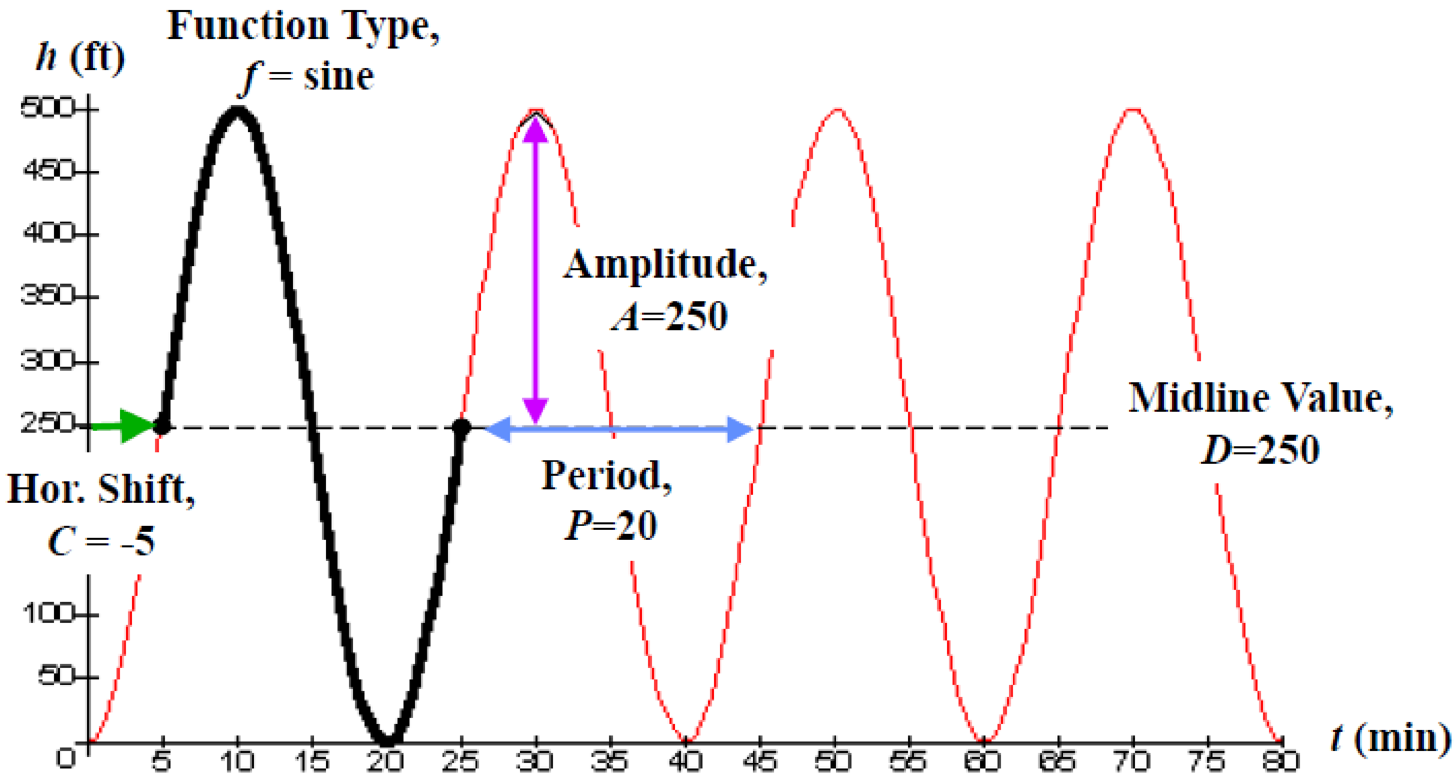
Definitions:

The **period** (or wavelength) of *f* is the length of one complete cycle.

The **midline** is the horizontal line midway between the function's minimum and maximum values.

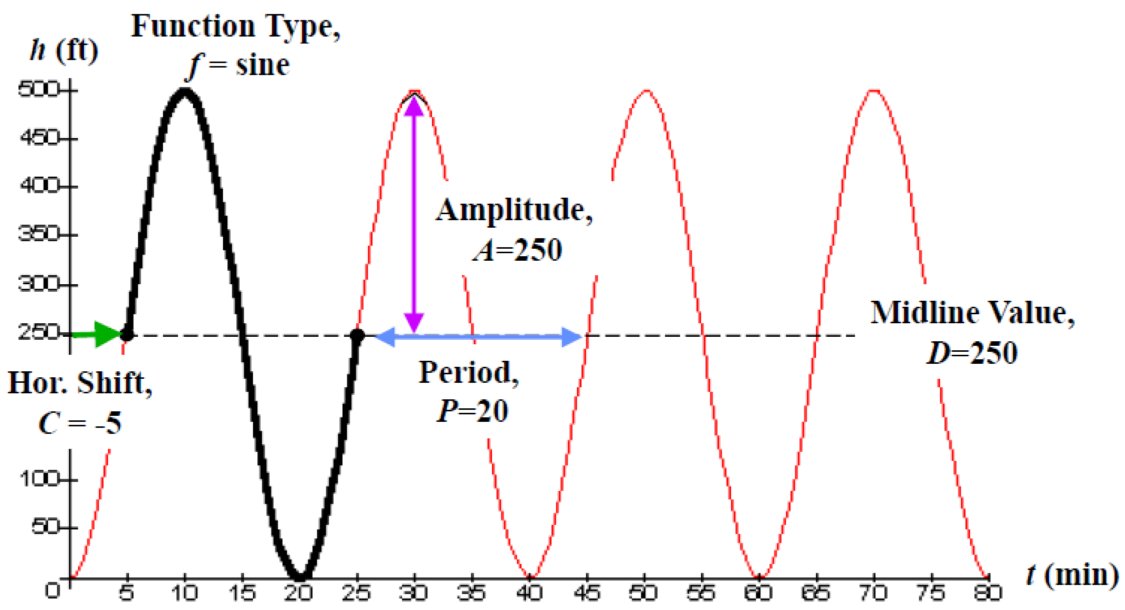
The **amplitude** is the distance between the function's maximum (or minimum) value and the midline.

The **phase (or horizontal) shift** is the number of units that the "start" of the cycle is away from being at the midline.



We can use these values to modify the basic cosine or sine function in order to model our Ferris wheel situation.

Try to come up with an equation that models our Ferris wheel situation. Check your equation on your graphing calculator.



Questions:

1. Compare your equation with another student or group. Is your solution unique?
2. What is your predicted height above the ground when you are 12 minutes into the ride?
3. Since the Ferris wheel rotates at a constant speed, when are you rising or falling the fastest? When are you rising or falling the slowest?

For now, let's simplify things and only consider the sine function. Similar relationships may be noticed for the cosine function.

Variations of the Sine Function

The parameters of the sine function

$$f(x) = a\sin(bx + h) + k$$

are interpreted as follows (we assume $b > 0$):

$|a|$ is the amplitude where $a < 0$ denotes reflection across the horizontal axis

b is the frequency, and $\frac{2\pi}{b}$ is the period

$\frac{|h|}{b}$ is the horizontal shift, right if $h < 0$ and left if $h > 0$

k is the vertical shift, up if $k > 0$ and down if $k < 0$

The applet linked below can help demonstrate how changes in these parameters affect the sinusoidal graph:

http://www.analyzemath.com/trigonometry/sine.htm